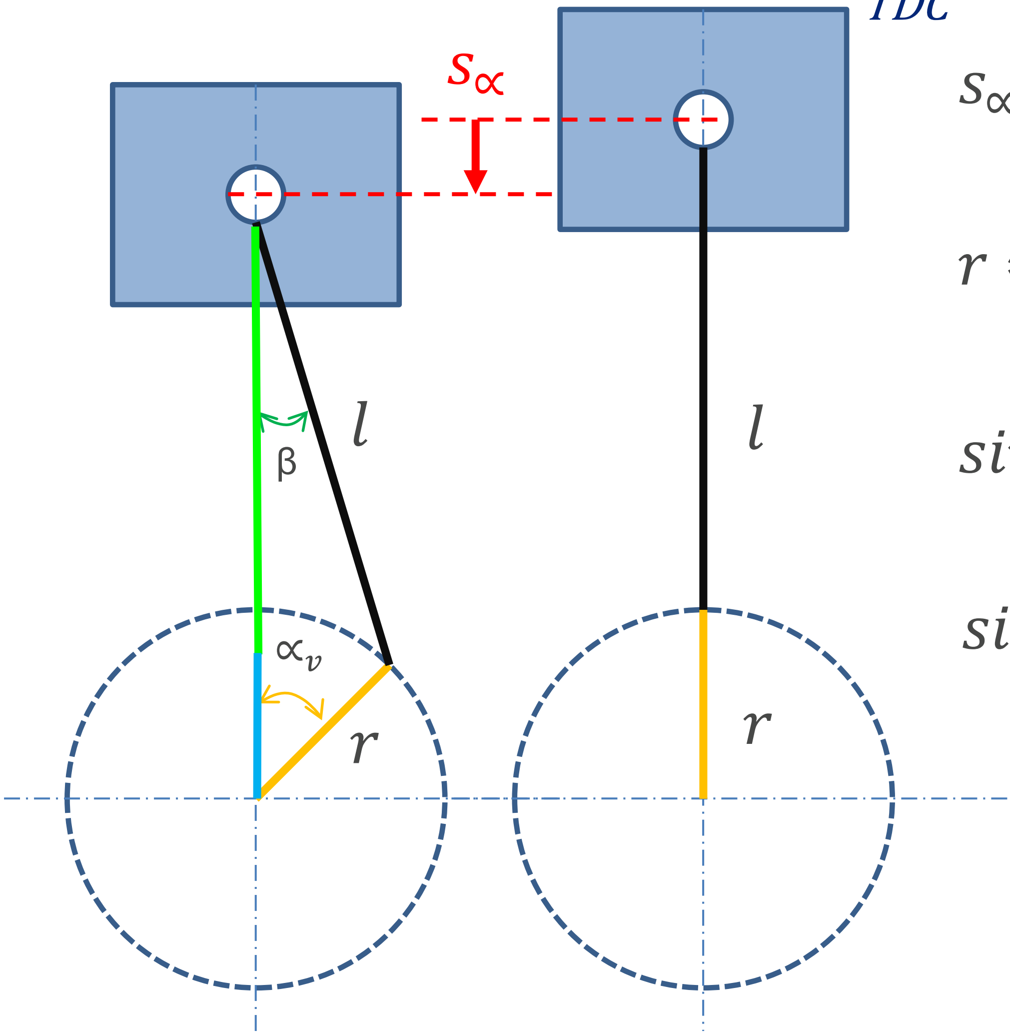
Engine Kinematics

AUTOPISTA

Pistón's position for "alpha"



$$s_\propto = r + l - l * cos\beta - r * cos \propto$$

Aplying trigonometry

$$r * sin \propto = l * sin\beta$$

Clearing sinus

$$sin\beta = \frac{r}{l} sin \propto = \lambda_s * sin \propto$$

Applying sin/cos law

$$sin^2\beta + cos^2\beta = 1$$

$$cos\beta = \sqrt{1 - sin^2\beta}$$

$$cos\beta = \sqrt{1 - {\lambda_s}^2 sin^2 \propto}$$

TDC = Top Dead Center

Pistón's position for "alpha"

$$s_{\propto} = r + l - l * cos\beta - r * cos \propto \qquad cos\beta = \sqrt{1 - \lambda_s{}^2 sin^2 \propto} \qquad \lambda_s = \frac{r}{l}$$

- Replacing

$$s_{\propto} = r + l - l * \sqrt{1 - \lambda_s^2 sin^2 \propto} - r * cos \propto$$

Piston's position equation for an angle Alpha.

- Applying Taylor's expansion and sin/cos identity of double angle

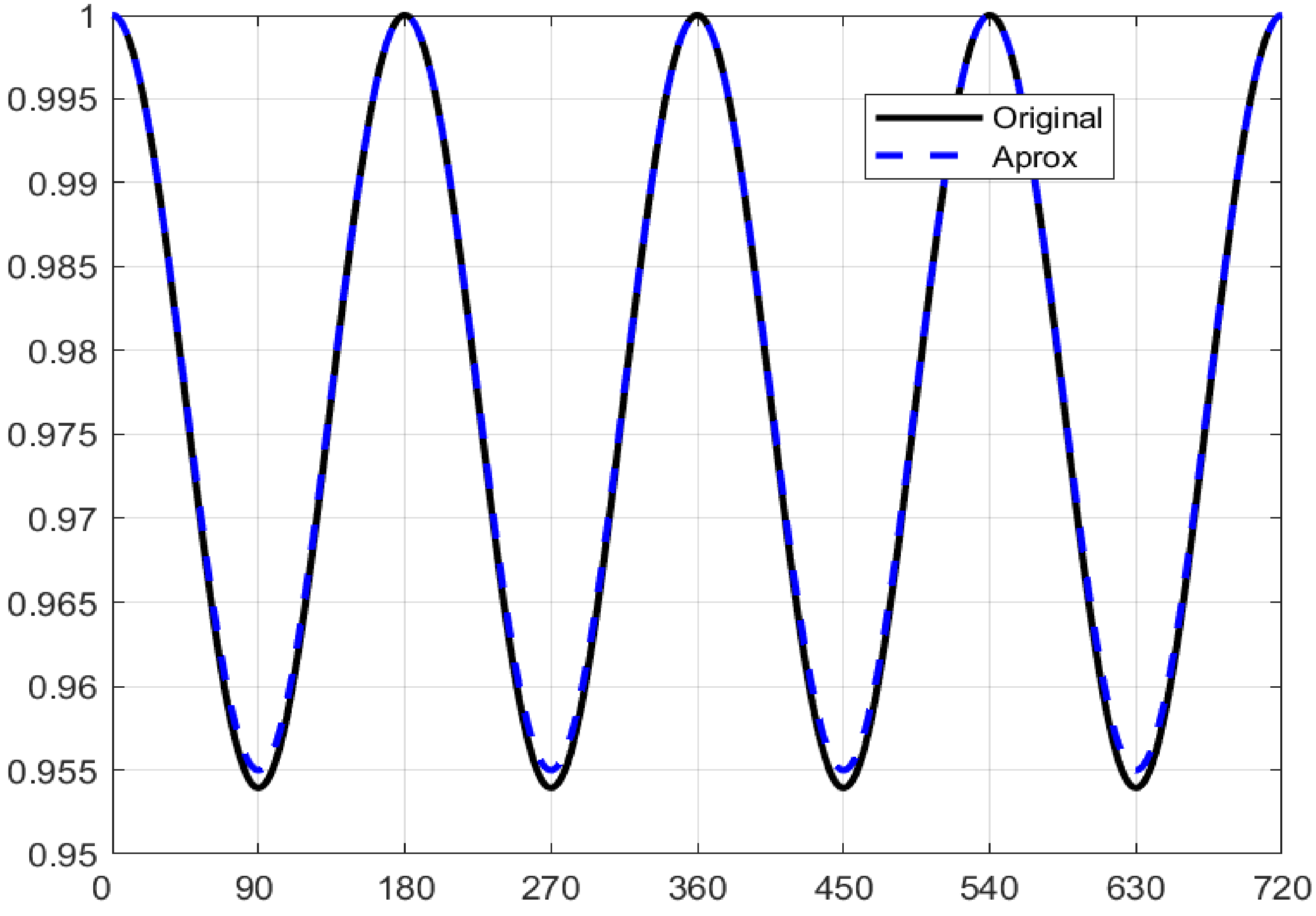
$$s_{\propto} \sim r\left[(1 - cos \propto) + \frac{\lambda_s}{4} * (1 - cos2 \propto)\right]$$

$$\sqrt{1 - \lambda_s^2 sin^2 \propto} \sim 1 - \frac{1}{2} * \lambda_s^2 sin^2 \propto$$

$$sin^2 \propto = \frac{1}{2} * (1 - cos2 \propto)$$

Taylor's expansion

We prove that: $\sqrt{1-\lambda_s^2 sin^2 \propto} \sim 1 - \frac{1}{2} * \lambda_s^2 sin^2 \propto$



Velocity and Acceleration

$$s_\propto \sim r\left[(1-\cos\propto)+\frac{\lambda_s}{4}*(1-\cos 2\propto)\right]$$

$$\lambda_s=\frac{r}{l}$$

- Derivative of the position -> Velocity

$$\dot{s_\propto}=\frac{ds_\propto}{d\propto}\cdot\frac{d\propto}{dt}=\frac{ds_\propto}{d\propto}*\omega$$

ω = angular speed = 2 pi * n [rad/s]*

$$\dot{s_\propto}\sim r*\omega\left(\sin\propto+\frac{\lambda_s}{2}*\sin 2\propto\right)$$

- Derivative of the velocity -> Acceleration

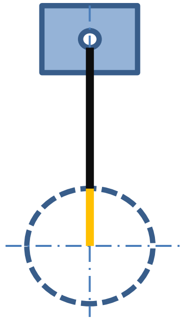
$$\ddot{s_\propto}=\frac{d^2 s_\propto}{d\propto^2}\cdot\frac{d^2\propto}{dt^2}=\frac{d^2 s_\propto}{d\propto^2}*\omega^2$$

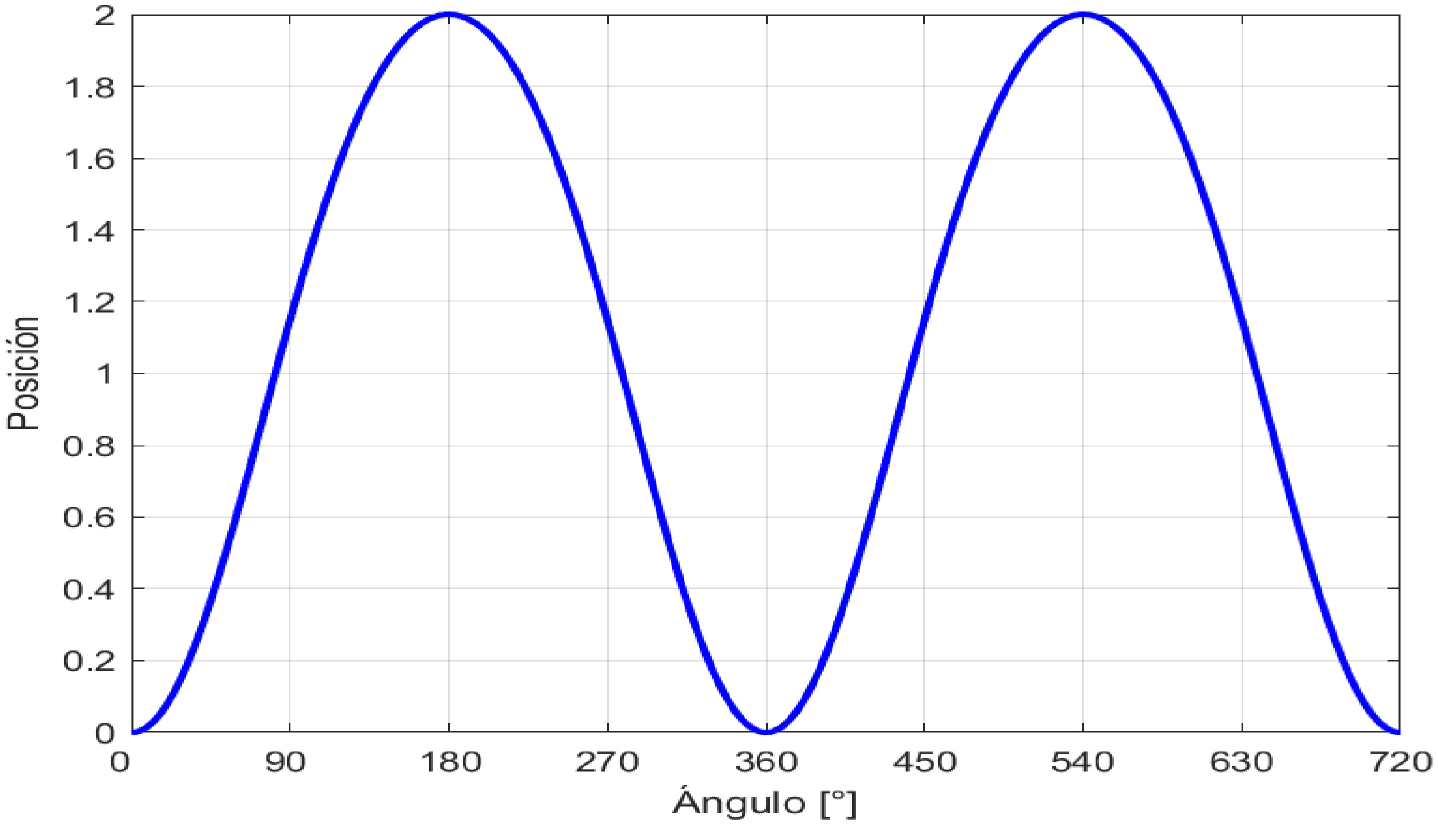
$$\ddot{s_\propto}\sim r*\omega^2(\cos\propto+\lambda_s*\cos 2\propto)$$

Position

Drawing a graph of the pistion’s position against “alpha”

$$s_{\propto} \sim r\left[(1-cos\propto)+\frac{\lambda_s}{4}*(1-cos2\propto)\right]$$

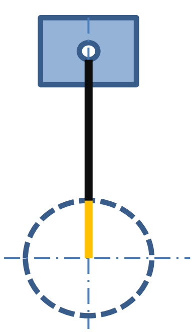


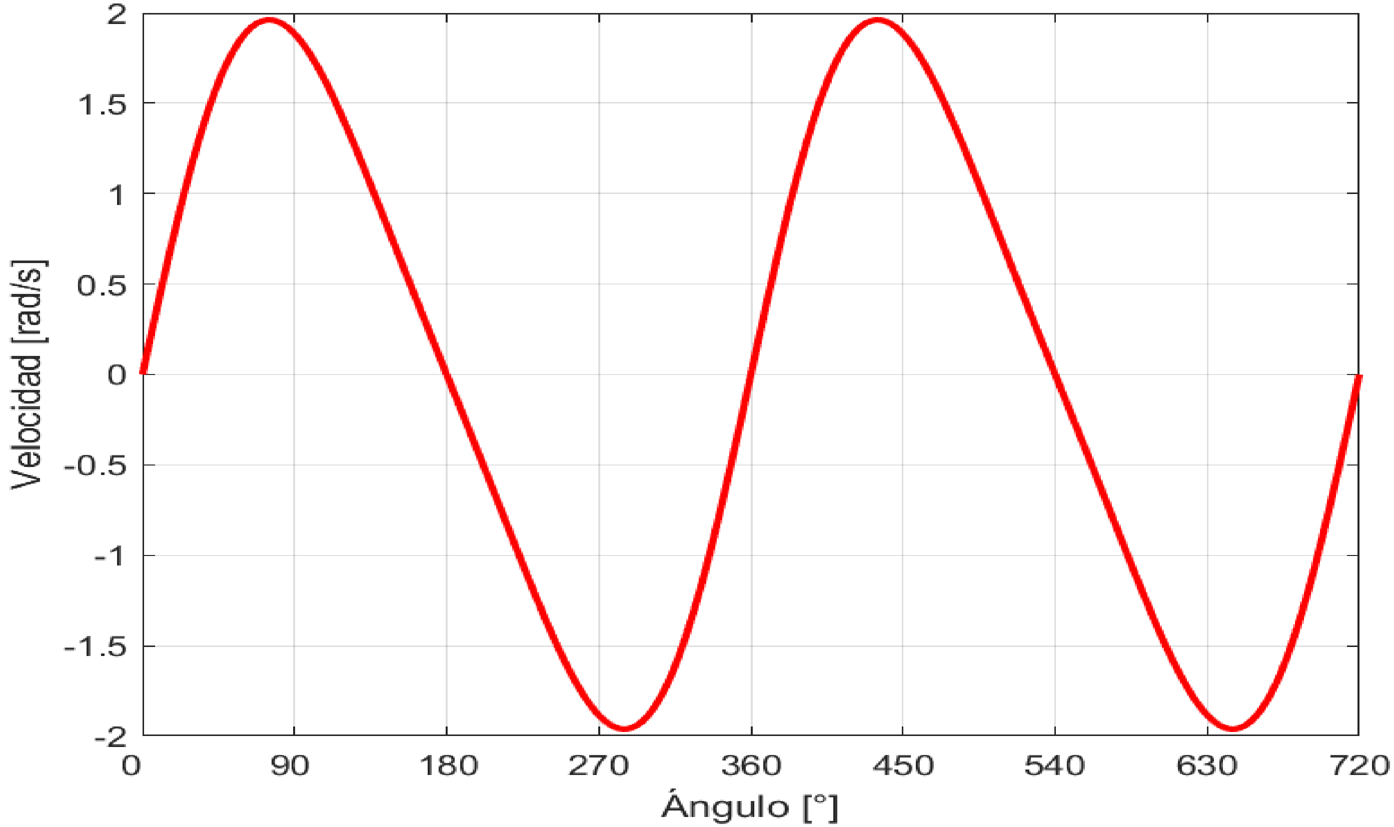


Velocity

Graph of the speed versus “alpha”

$$\dot{s}_{\propto} \sim r * \omega(sin \propto + \frac{\lambda_s}{2} * sin2 \propto)$$

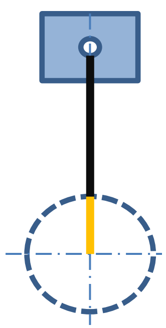


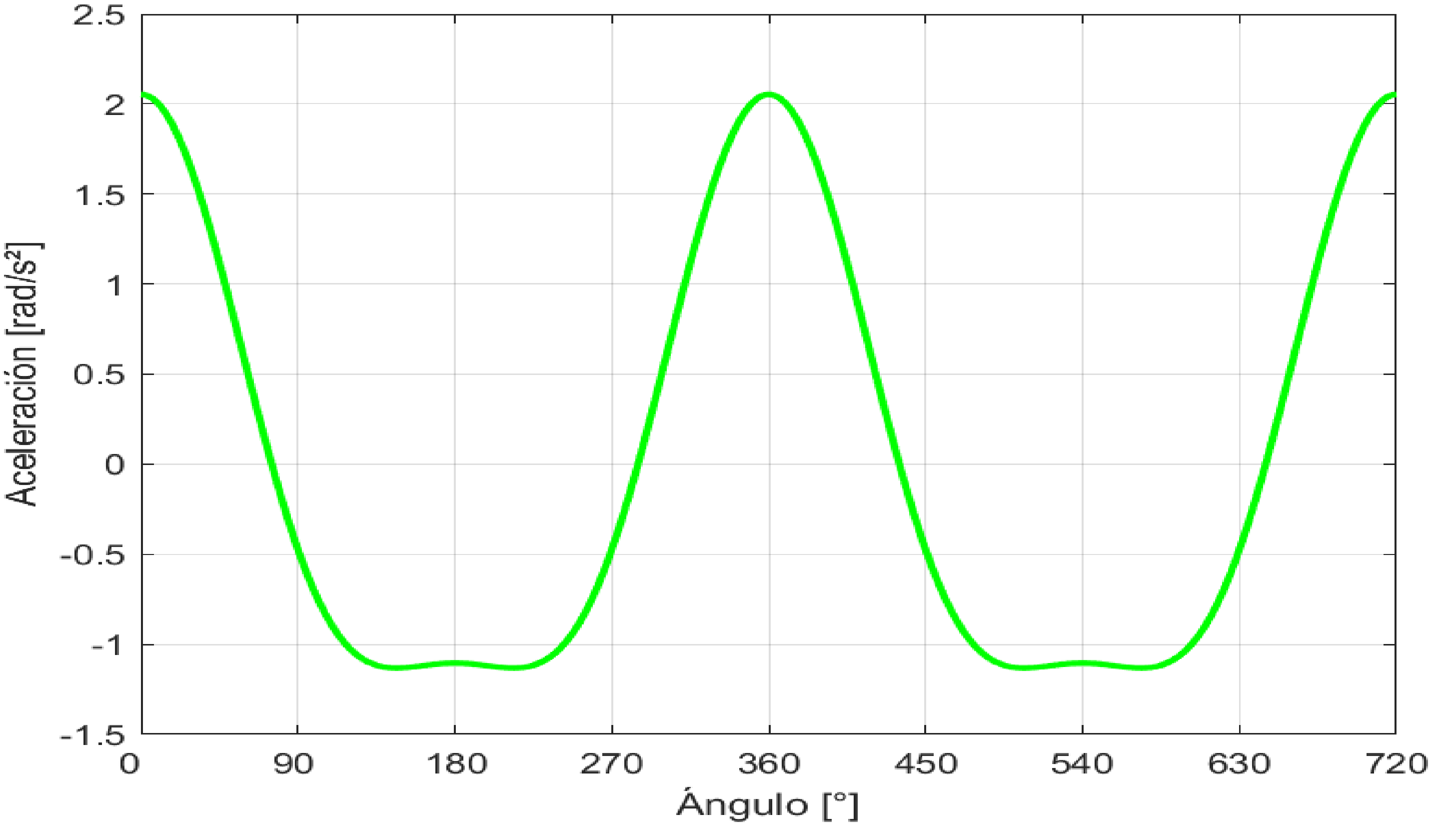


Acceleration

Acceleration’s graph versus “alpha”

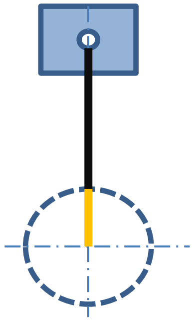
$\ddot{s}_{\propto} \sim r * \omega^2 (\cos \propto + \lambda_s * \cos 2 \propto)$

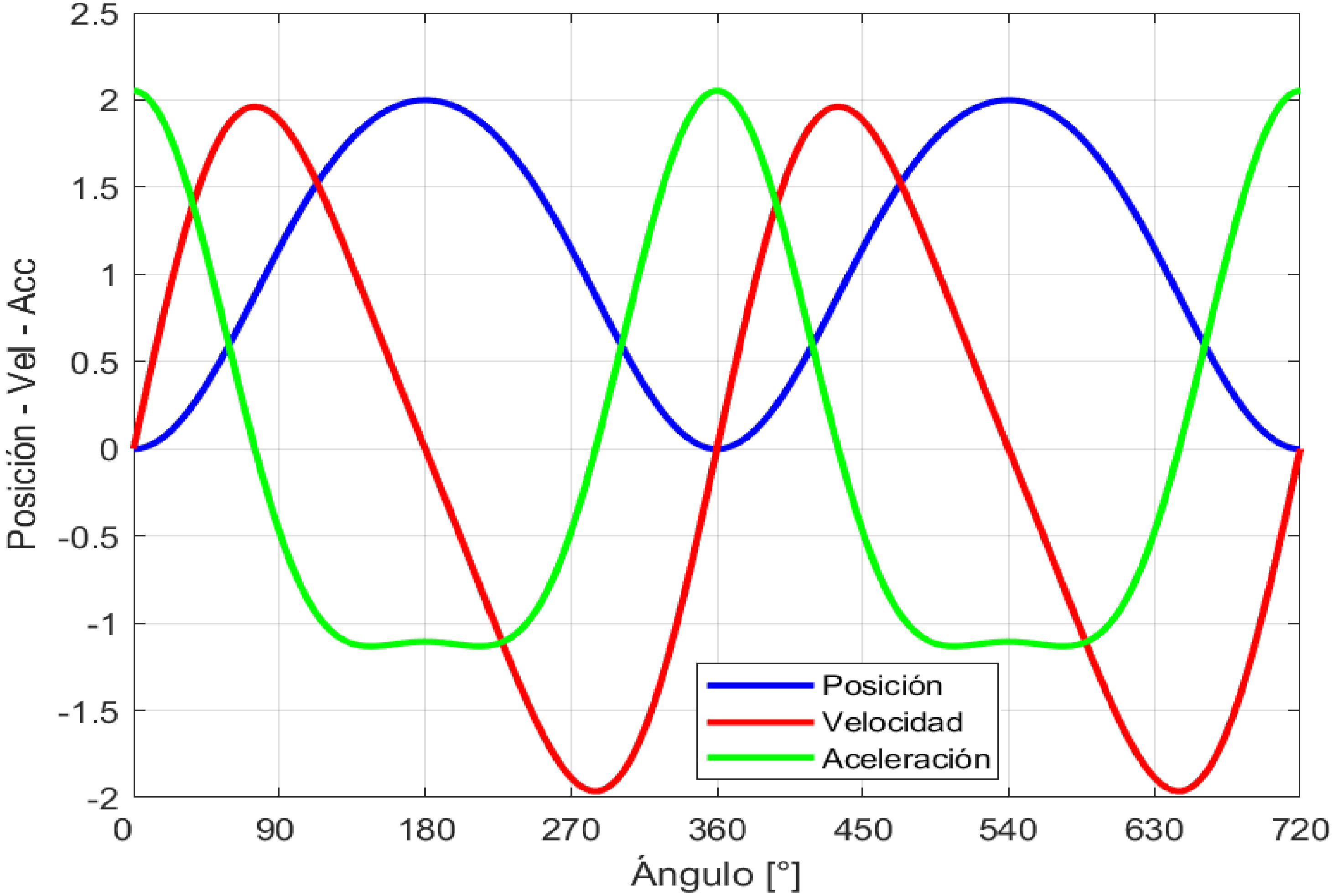




Position – Velocity – Acceleration

Graphing all together versus "Alpha"





Summary

- Geometry and definition of the angle “alpha”.
- Equations for position, speed and acceleration.
- Taylor’s expansión review.
- With the acceleration’s equation, one can calculate:
  - Mass force (Fm = mass * acceleration) and
  - Partial Torque of the crankshaft -> The gas forces are not yet considered Fg.